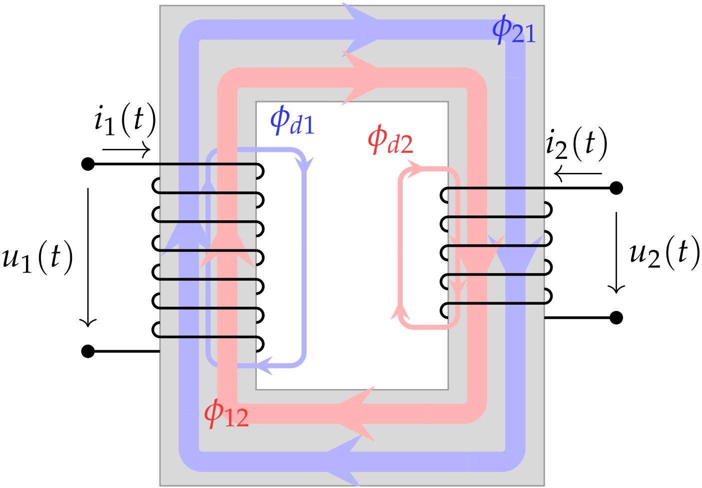
Transformadores

Teoría de Circuitos II

Oscar Perpiñán Lamigueiro

Recordatorio



$$L_1 = N_1 \frac{\phi_{11}}{i_1}$$

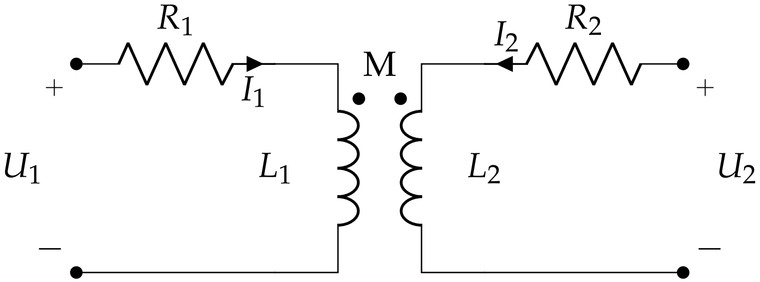
$$L_2 = N_2 \frac{\phi_{22}}{i_2}$$

$$M = N_1 \frac{\phi_{12}}{i_2}$$

$$= N_2 \frac{\phi_{21}}{i_1}$$

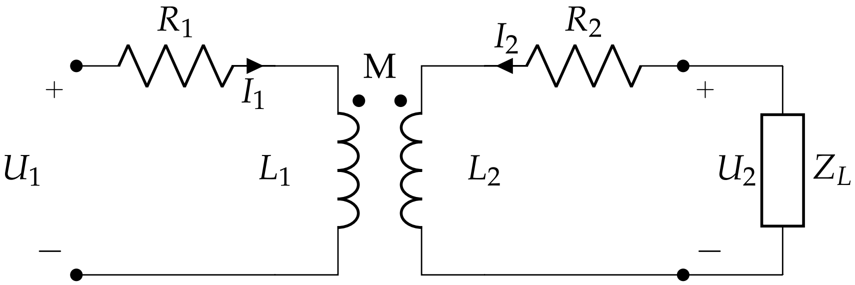
$$M = k\sqrt{L_1 \cdot L_2}$$

Ecuaciones del Transformador Real



$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + (R_2 + j\omega L_2) \cdot \bar{I}_2$$

Ejemplo: impedancia de entrada desde primario



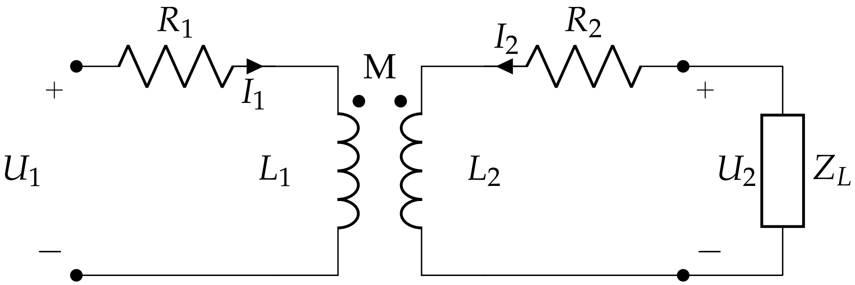
Ecuaciones del transformador

$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + (R_2 + j\omega L_2) \cdot \bar{I}_2$$

Ecuación de la carga

$$\overline{U}_2 = -\bar{I}_2 \cdot \overline{Z}_L$$

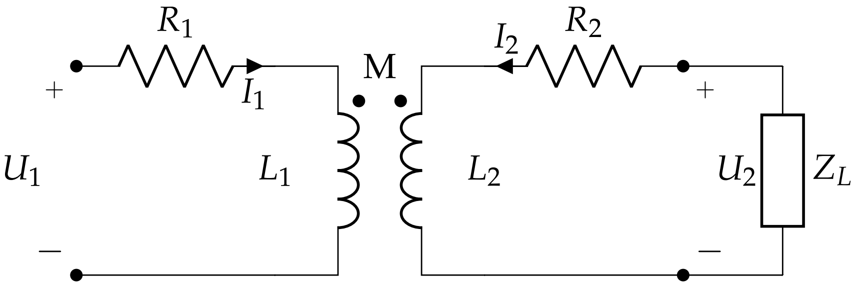
Ejemplo: impedancia de entrada desde primario



Combinando la ecuación del secundario con la ecuación de la carga:

$$\bar{I}_2 = -\frac{j\omega M}{(R_2 + j\omega L_2) + \overline{Z}_L} \cdot \bar{I}_1$$

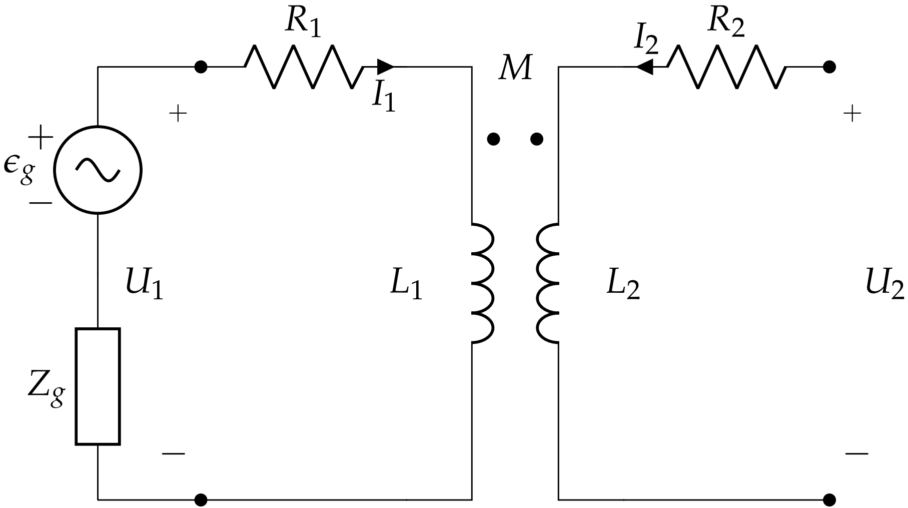
Ejemplo: impedancia de entrada desde primario



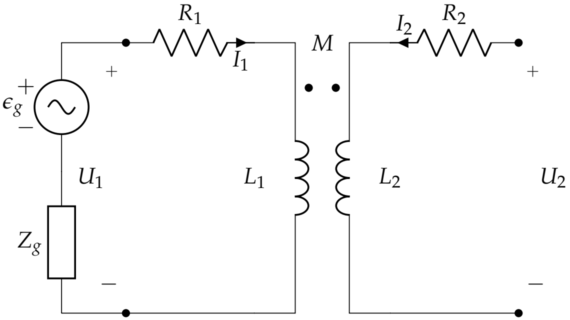
Combinando con la ecuación del primario:

$$\overline{Z}_{in} = \frac{\overline{U}_1}{\overline{I}_1} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2) + \overline{Z}_L} = \boxed{\overline{Z}_1 + \frac{\omega^2 M^2}{\overline{Z}_2 + \overline{Z}_L}}$$

Ejemplo: Equivalente de Thévenin desde secundario



Tensión de Thévenin



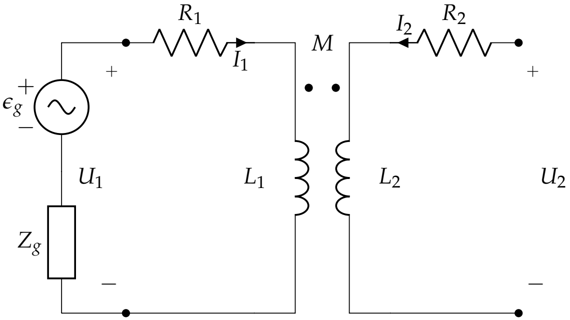
Ecuación del generador:

$$\overline{U}_1 = \overline{\epsilon}_g - \overline{I}_1 \cdot \overline{Z}_g$$

Ecuaciones del transformador ($\overline{I}_2 = 0$)

$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \overline{I}_1$$
$$\overline{U}_2 = j\omega M \cdot \overline{I}_1$$

Tensión de Thévenin



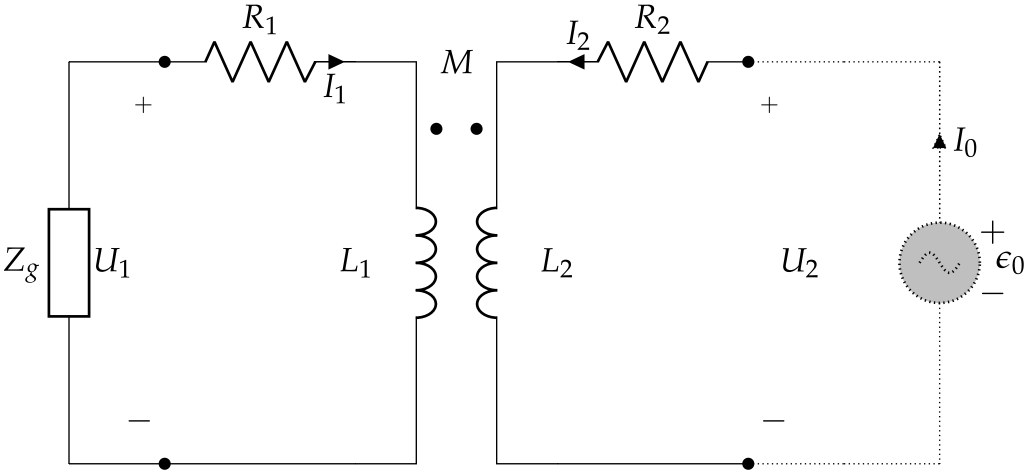
Despejamos I_1:

$$\overline{I}_1 = \frac{\overline{\epsilon_g}}{\overline{Z}_1 + \overline{Z}_g}$$

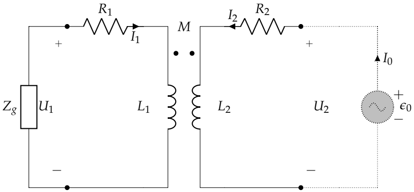
Tensión en abierto:

$$\overline{U}_2 = \boxed{\overline{\epsilon}_{th} = \frac{j\omega M}{\overline{Z}_1 + \overline{Z}_g} \cdot \overline{\epsilon}_g}$$

Impedancia de Thévenin



Impedancia de Thévenin



Ecuaciones del transformador:

$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_0$$
$$\bar{\epsilon}_0 = j\omega M \cdot \bar{I}_1 + (R_2 + j\omega L_2) \cdot \bar{I}_0$$

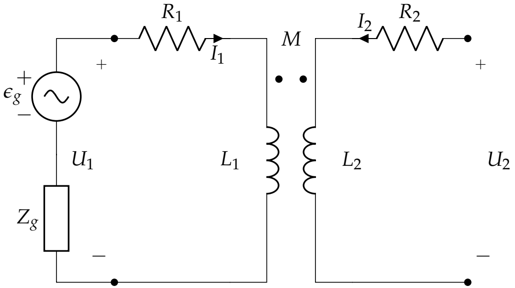
Ecuación de la impedancia:

$$\overline{U}_1 = -\overline{Z}_g \cdot \bar{I}_1$$

Impedancia de Thévenin:

$$\boxed{\overline{Z}_{th} = \frac{\bar{\epsilon}_0}{\bar{I}_0} = \overline{Z}_2 + \frac{\omega^2 M^2}{\overline{Z}_1 + \overline{Z}_g}}$$

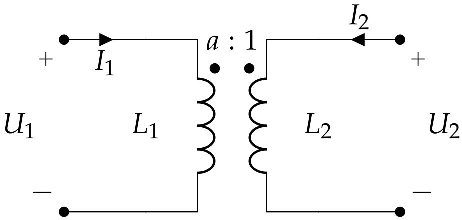
Resumen: Equivalente de Thévenin



$$\overline{\epsilon}_{th} = \frac{j\omega M}{\overline{Z}_1 + \overline{Z}_g} \cdot \overline{\epsilon}_g$$

$$\overline{Z}_{th} = \frac{\overline{\epsilon}_0}{\overline{I}_0} = \overline{Z}_2 + \frac{\omega^2 M^2}{\overline{Z}_1 + \overline{Z}_g}$$

Definición



Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = 0$$

El acoplamiento es perfecto.

$$k = 1 \rightarrow \begin{cases} \phi_{12} & = \phi_{22} \\ \phi_{21} & = \phi_{11} \end{cases}$$

Relación de Transformación

Retomamos las ecuaciones de $M_{12} = M_{21} = M$:

$$N_1 \frac{\phi_{12}}{i_2} = N_2 \frac{\phi_{21}}{i_1}$$

Con la condición $k = 1$ escribimos:

$$N_1 \frac{\phi_{22}}{i_2} = N_2 \frac{\phi_{11}}{i_1}$$

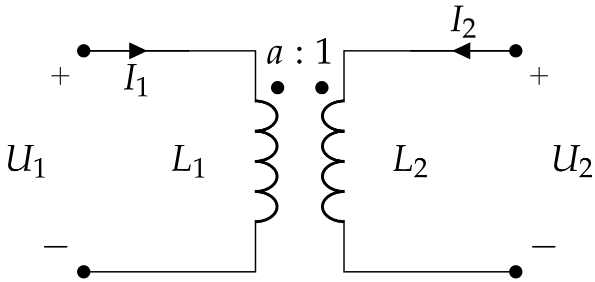
Y con las definiciones de L_1 y L_2:

$$N_1 \frac{L_2}{N_2} = N_2 \frac{L_1}{N_1}$$

Obtenemos la relación de transformación:

$$\boxed{\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 = a^2}$$

Ecuaciones del Transformador Perfecto



$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Relación de Tensiones

Dividiendo las ecuaciones:

$$\frac{\overline{U}_1}{\overline{U}_2} = \frac{j\omega L_1 \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2}{j\omega M \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2}$$

Empleando la relación de transformación:

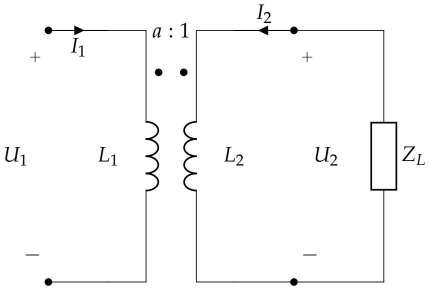
$$\frac{L_1}{L_2} = a^2 \rightarrow \begin{cases} L_1 & = a^2 \cdot L_2 \\ M & = a \cdot L_2 \end{cases}$$

Obtenemos:

$$\frac{\overline{U}_1}{\overline{U}_2} = \frac{a^2 L_2 \cdot \bar{I}_1 + a L_2 \cdot \bar{I}_2}{a L_2 \cdot \bar{I}_1 + L_2 \cdot \bar{I}_2}$$

$$\boxed{\frac{\overline{U}_1}{\overline{U}_2} = a = \frac{N_1}{N_2}}$$

Ejemplo: Impedancia de Entrada



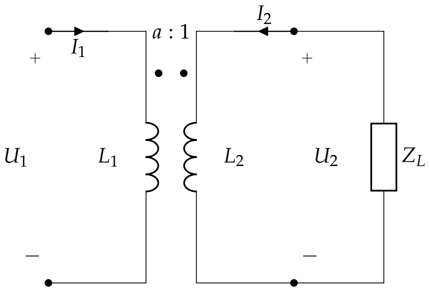
Ecuaciones del transformador:

$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Ecuación de la impedancia:

$$\overline{U}_2 = -\overline{Z}_L \cdot \bar{I}_2$$

Ejemplo: Impedancia de Entrada



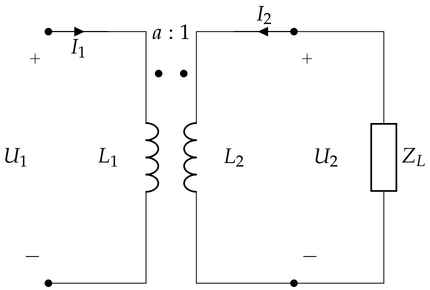
Despejamos I_2:

$$\bar{I}_2 = -\frac{j\omega M}{j\omega L_2 + \overline{Z}_L} \cdot \bar{I}_1$$

Y sustituimos:

$$\overline{Z}_{in} = \frac{\overline{U}_1}{\bar{I}_1} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \overline{Z}_L}$$

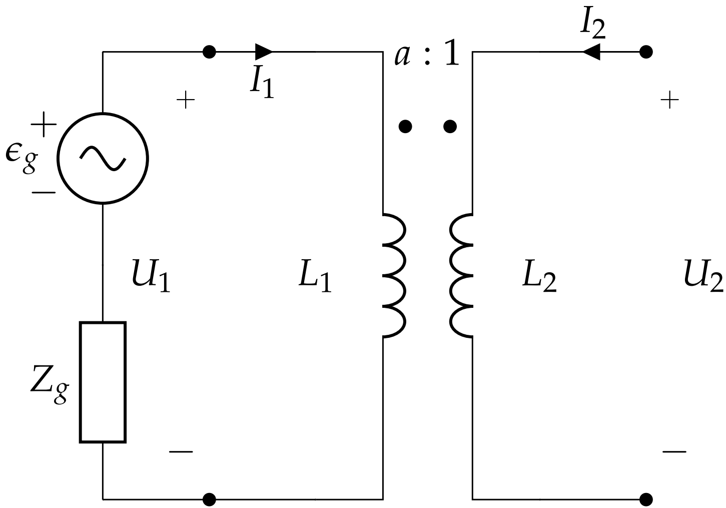
Ejemplo: Impedancia de Entrada



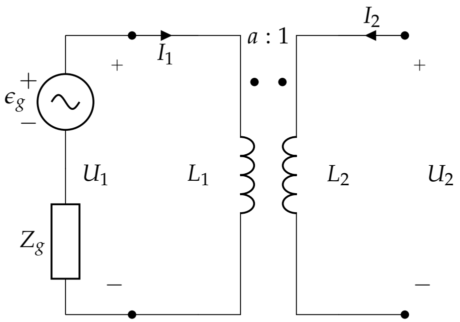
Teniendo en cuenta la relación entre L_1, L_2 y M:

$$\overline{Z}_{in} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \overline{Z}_L} = \frac{j\omega L_1 \overline{Z}_L}{j\omega L_2 + \overline{Z}_L} \rightarrow \boxed{\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L} = \frac{j\omega L_1 \cdot a^2 \cdot \overline{Z}_L}{j\omega L_1 + a^2 \cdot \overline{Z}_L}}$$

Ejemplo: Equivalente de Thévenin desde secundario



Tensión de Thévenin



Ecuación del generador:

$$\overline{U}_1 = \overline{\epsilon}_g - \overline{I}_1 \cdot \overline{Z}_g$$

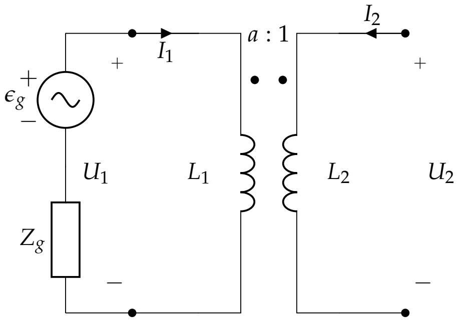
Ecuaciones del transformador ($\overline{I}_2 = 0$)

$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1$$
$$\overline{U}_2 = j\omega M \cdot \overline{I}_1$$

Despejamos I_1:

$$\overline{I}_1 = \frac{\overline{\epsilon_g}}{\overline{Z}_g + j\omega L_1}$$

Tensión de Thévenin



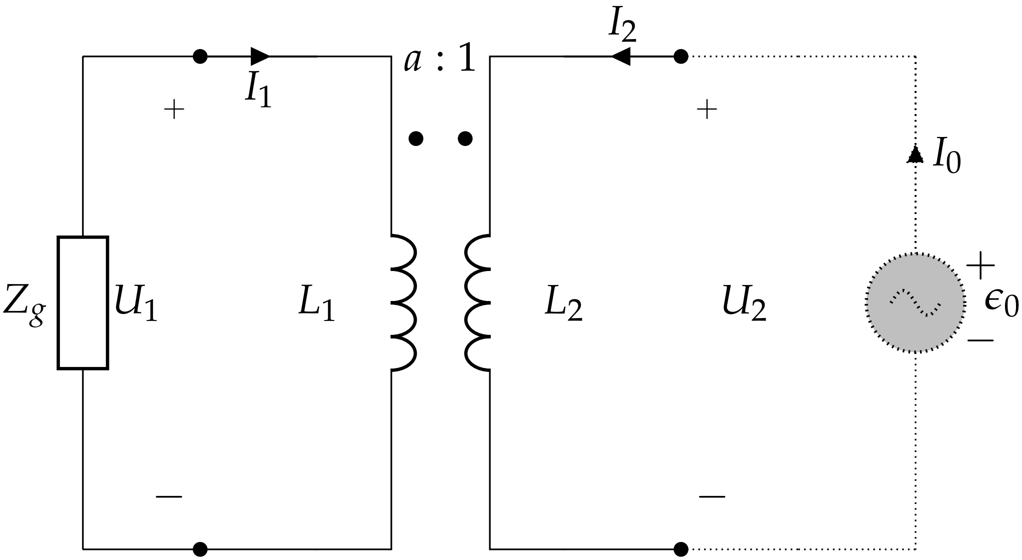
Tensión en abierto:

$$\overline{\epsilon}_{th} = \overline{U}_2 = \frac{j\omega M}{j\omega L_1 + \overline{Z}_g} \cdot \overline{\epsilon}_g$$

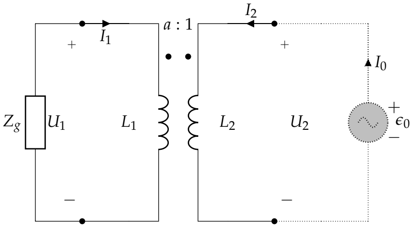
Teniendo en cuenta que $M = L_1/a$:

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Impedancia de Thévenin



Impedancia de Thévenin



Ecuaciones del transformador:

$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega M \cdot \overline{I}_0$$
$$\overline{\epsilon}_0 = j\omega M \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_0$$

Ecuación de la impedancia:

$$\overline{U}_1 = -\overline{Z}_g \cdot \overline{I}_1$$

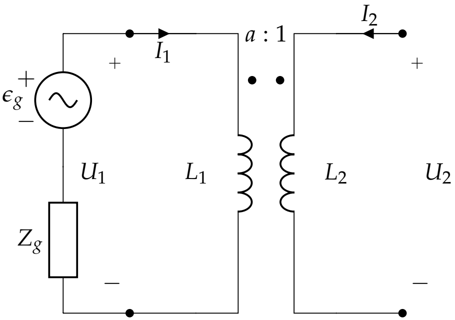
Impedancia de Thévenin:

$$\overline{Z}_{th} = \frac{\overline{\epsilon}_0}{\overline{I}_0} = j\omega L_2 + \frac{\omega^2 M^2}{j\omega L_1 + \overline{Z}_g}$$

Con $L_2 = L_1/a^2$ y $M = L_1/a$:

$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

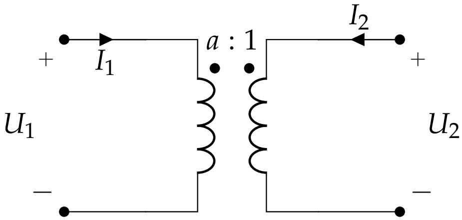
Resumen: Equivalente de Thévenin



$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Definición



Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = 0$$

El acoplamiento es perfecto.

$$k = 1$$

Las bobinas tienen un número muy elevado de espiras.

$$N_1 \to \infty$$

$$N_2 \to \infty$$

El flujo en cada bobina es nulo

Para que las tensiones inducidas sean finitas…

$$\overline{U}_1 = N_1\overline{\phi}_1$$
$$\overline{U}_2 = N_2\overline{\phi}_2$$

…los flujos (fasoriales) que los atraviesan deben ser nulos.

$$\overline{\phi}_1 \rightarrow 0$$
$$\overline{\phi}_2 \rightarrow 0$$

Siendo:

$$\overline{\phi}_1 = \overline{\phi}_{11} + \overline{\phi}_{12}$$
$$\overline{\phi}_2 = \overline{\phi}_{22} + \overline{\phi}_{21}$$

El flujo mutuo es nulo

Teniendo en cuenta que el acoplamiento es perfecto, $k = 1$:

$$\left.\begin{aligned} \phi_{12} &= \phi_{22} \\ \phi_{21} &= \phi_{11} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} 0 &= \overline{\phi}_{21} + \overline{\phi}_{12} \\ 0 &= \overline{\phi}_{12} + \overline{\phi}_{21} \end{aligned}\right.$$

O también:

$$\boxed{\overline{\phi}_{11} + \overline{\phi}_{22} = 0}$$

Relación de Transformación

Hemos obtenido:

$$\overline{\phi}_{11} + \overline{\phi}_{22} = 0$$

Con las definiciones de L_1, L_2:

$$L_1 = N_1 \frac{\phi_{11}}{I_1}; \quad L_2 = N_2 \frac{\phi_{22}}{I_2}$$

Podemos escribir:

$$\frac{L_1 \bar{I}_1}{N_1} + \frac{L_2 \bar{I}_2}{N_2} = 0$$

Y con la relación entre ambas obtenemos*:

$$L_1 = L_2 \cdot \left(\frac{N_1}{N_2}\right)^2 \rightarrow \frac{N_1 L_2 \bar{I}_1}{N_2^2} + \frac{L_2 \bar{I}_2}{N_2} = 0 \rightarrow \boxed{\frac{\bar{I}_1}{\bar{I}_2} = -\frac{1}{a} = -\frac{N_2}{N_1}}$$

*Si las dos corrientes van en el mismo sentido, la ecuación tendrá un signo positivo.

Un transformador ideal no consume potencia

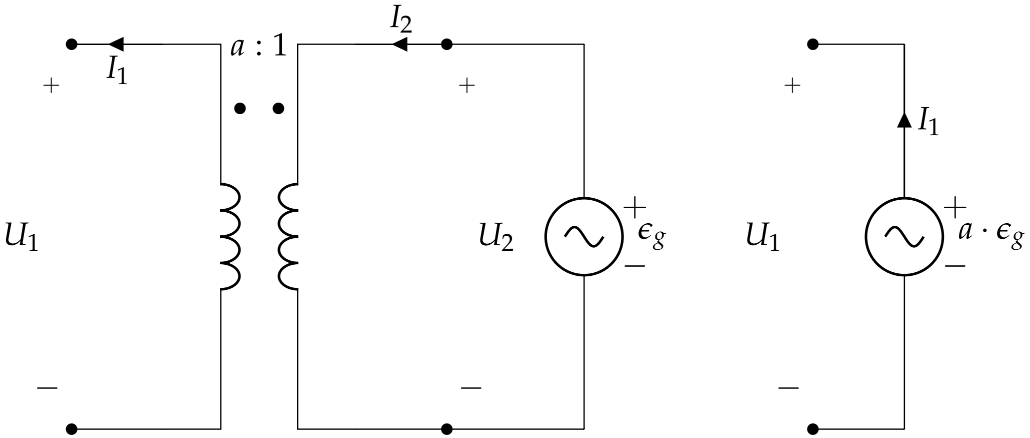
$$\overline{S}_1 = \overline{U}_1 \cdot \overline{I}_1^*$$

$$\overline{S}_2 = \overline{U}_2 \cdot \overline{I}_2^*$$

$$\overline{U}_2 \cdot \overline{I}_2^* = \frac{1}{a} \cdot \overline{U}_1 \cdot a \cdot \overline{I}_1^* = \overline{S}_1$$

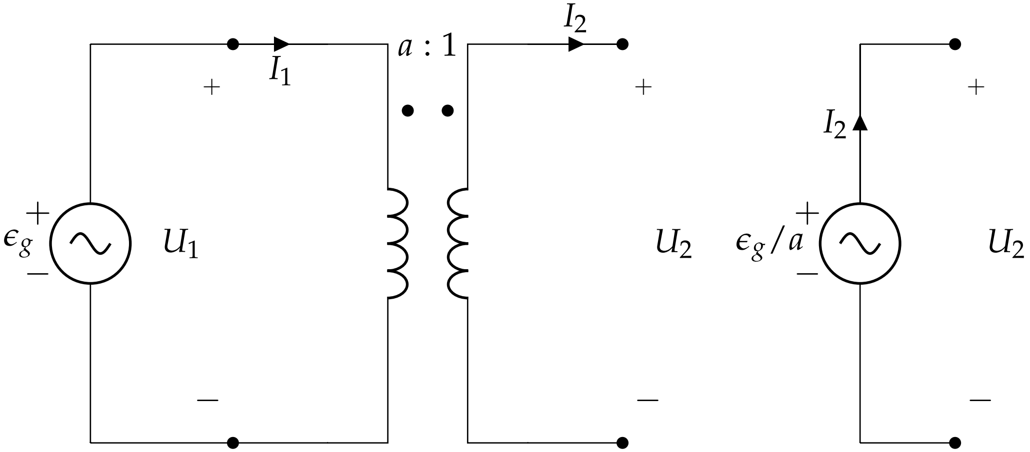
$$\boxed{\overline{S}_1 = \overline{S}_2} \quad \boxed{P_1 = P_2} \quad \boxed{Q_1 = Q_2}$$

Fuente de Tensión de Secundario a Primario



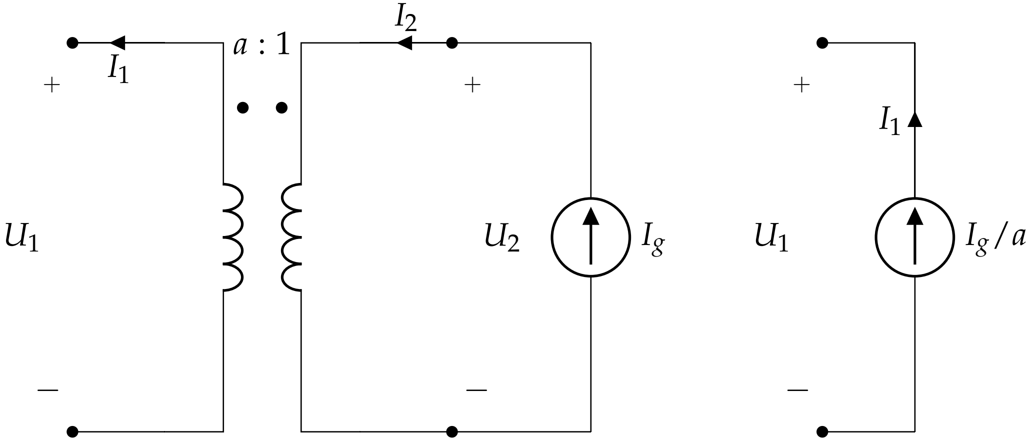
$$\overline{U}_1 = a \cdot \overline{U}_2 \rightarrow \boxed{\overline{\epsilon}_{g1} = a \cdot \overline{\epsilon}_g}$$

Fuente de Tensión de Primario a Secundario



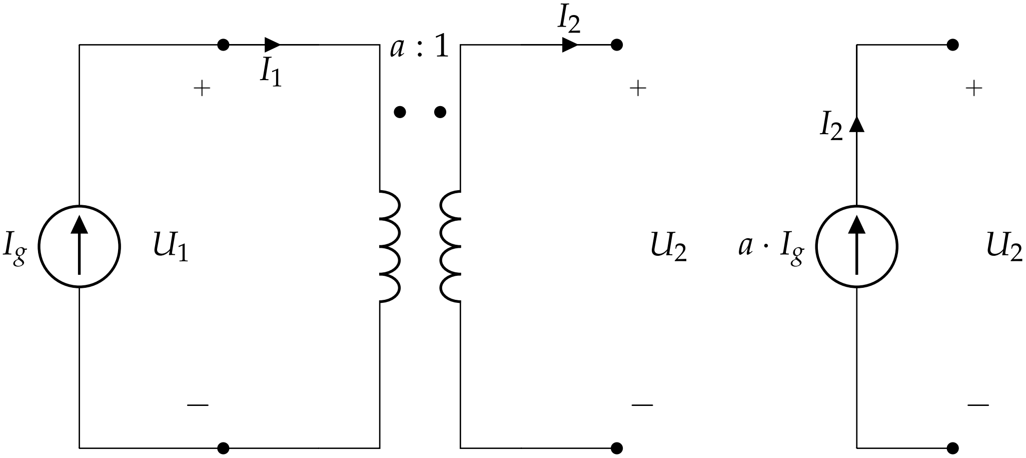
$$\overline{U}_2 = \overline{U}_1/a \rightarrow \boxed{\overline{\epsilon}_{g2} = \overline{\epsilon}_g/a}$$

Fuente de Corriente de Secundario a Primario



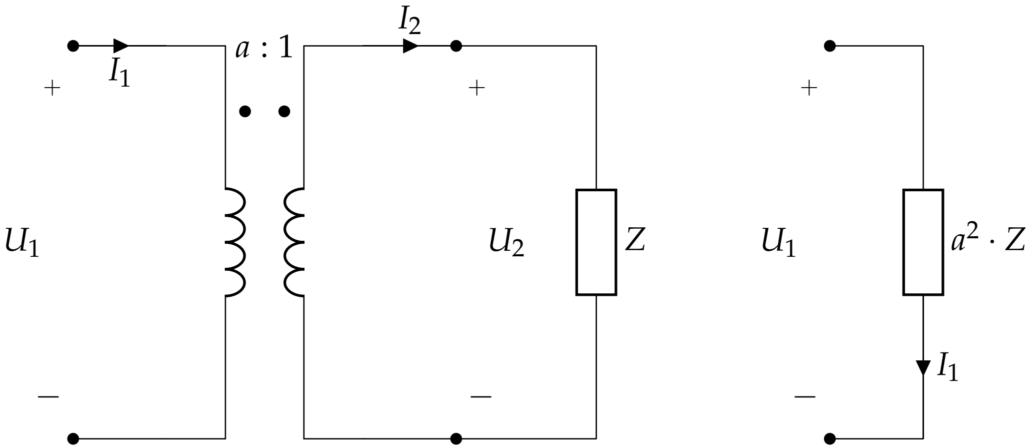
$$\bar{I}_1 = \bar{I}_2/a \rightarrow \boxed{\bar{I}_{g1} = \bar{I}_g/a}$$

Fuente de Corriente de Primario a Secundario



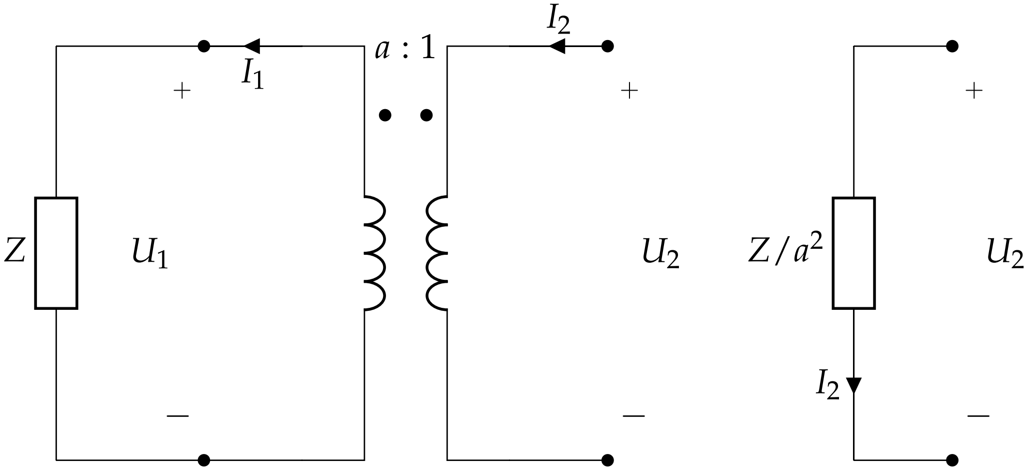
$$\bar{I}_2 = a \cdot \bar{I}_1 \rightarrow \boxed{\bar{I}_{g2} = a \cdot \bar{I}_g}$$

Impedancia de Secundario a Primario



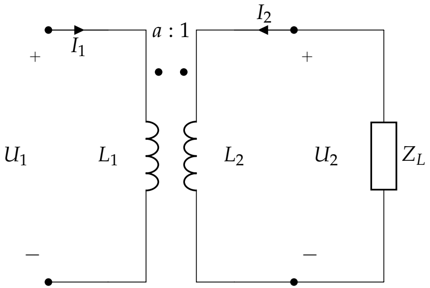
$$\left.\begin{aligned}\overline{U}_1 &= a \cdot \overline{U}_2 \\ \overline{I}_1 &= \overline{I}_2 / a\end{aligned}\right\} \rightarrow \boxed{\overline{Z}_1 = \frac{\overline{U}_1}{\overline{I}_1} = a^2 \cdot \overline{Z}}$$

Impedancia de Primario a Secundario



$$\left.\begin{aligned}\overline{U}_2 &= \overline{U}_1/a\\ \overline{I}_2 &= a\cdot\overline{I}_1\end{aligned}\right\} \rightarrow \boxed{\overline{Z}_2 = \frac{\overline{U}_2}{\overline{I}_2} = \overline{Z}/a^2}$$

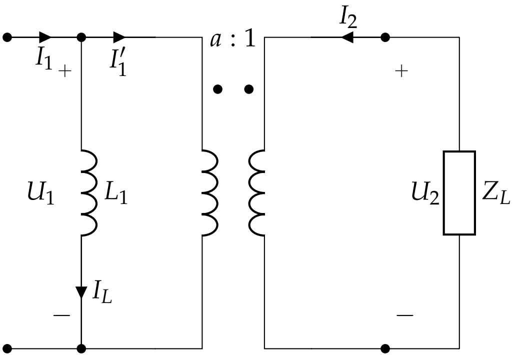
Recordatorio: impedancia de entrada de un T. Perfecto

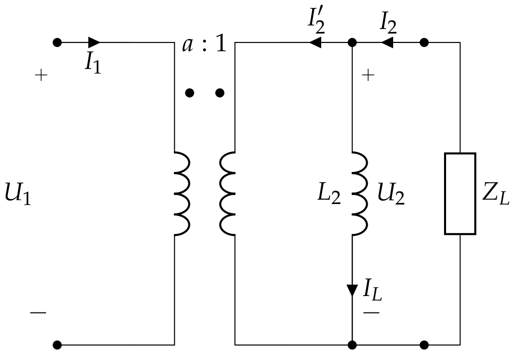


$$\overline{Z}_{in} = \frac{j\omega L_1 \cdot (a^2 \overline{Z}_L)}{j\omega L_1 + (a^2 \cdot \overline{Z}_L)}$$

$$\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L}$$

Circuito equivalente con transformador ideal

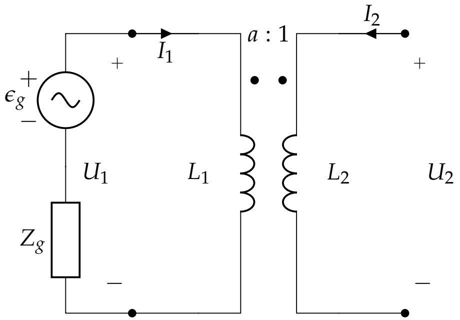




$$\overline{Z}_{in} = \frac{j\omega L_1 \cdot (a^2 \overline{Z}_L)}{j\omega L_1 + (a^2 \cdot \overline{Z}_L)}$$

$$\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L}$$

Recordatorio: Equivalente de Thévenin



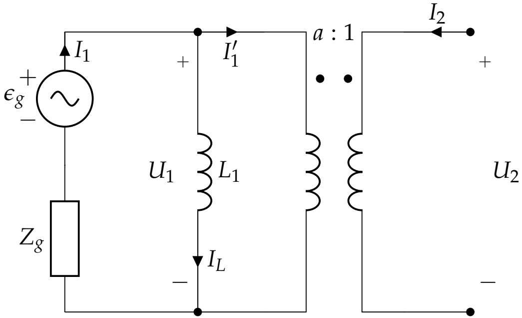
$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Equivalente en primario con transformador ideal

$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

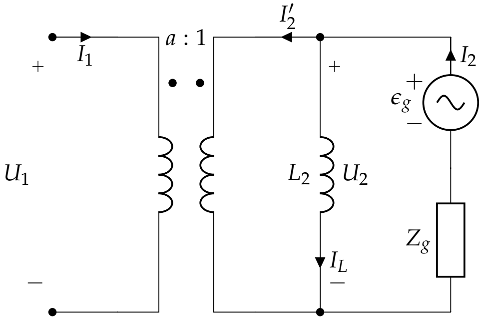
$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$



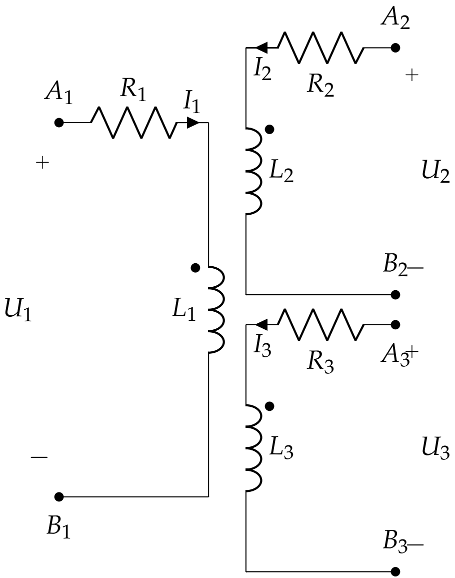
Equivalente en secundario con transformador ideal

$$\overline{Z}_{th} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_g}{j\omega L_2 + \overline{Z}_g}$$

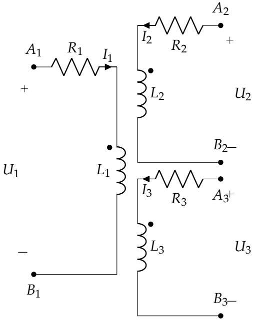
$$\overline{\epsilon}_{th} = a \cdot \left(\frac{j\omega L_2}{j\omega L_2 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$



Transformador Real de Varios Devanados



Ecuaciones del Transformador Real

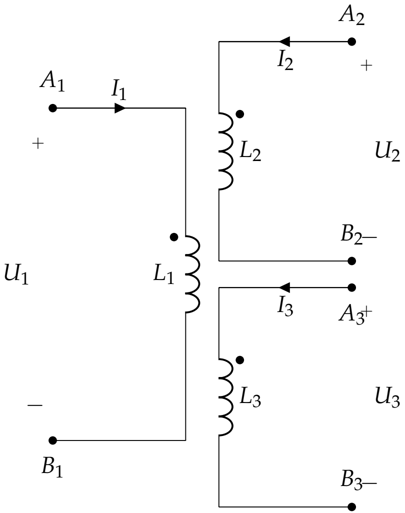


$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \bar{I}_1 + j\omega M_{12} \cdot \bar{I}_2 + j\omega M_{13} \cdot \bar{I}_3$$

$$\overline{U}_2 = j\omega M_{12} \cdot \bar{I}_1 + (R_2 + j\omega L_2) \cdot \bar{I}_2 + j\omega M_{23} \cdot \bar{I}_3$$

$$\overline{U}_3 = j\omega M_{13} \cdot \bar{I}_1 + j\omega M_{12} \cdot \bar{I}_2 + (R_3 + j\omega L_3) \cdot \bar{I}_3$$

Transformador Perfecto



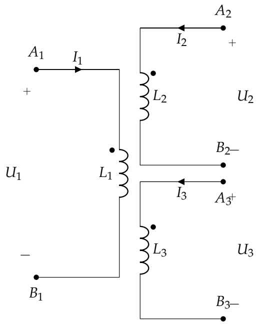
Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = R_3 = 0$$

El acoplamiento es perfecto.

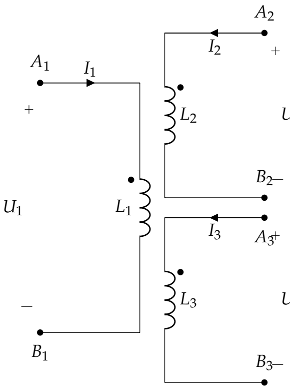
$$k_{12} = k_{13} = k_{23} = 1$$

Ecuaciones del Transformador Perfecto



$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega M_{12} \cdot \overline{I}_2 + j\omega M_{13} \cdot \overline{I}_3$$
$$\overline{U}_2 = j\omega M_{12} \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2 + j\omega M_{23} \cdot \overline{I}_3$$
$$\overline{U}_3 = j\omega M_{13} \cdot \overline{I}_1 + j\omega M_{12} \cdot \overline{I}_2 + j\omega L_3 \cdot \overline{I}_3$$

Relaciones de Transformación



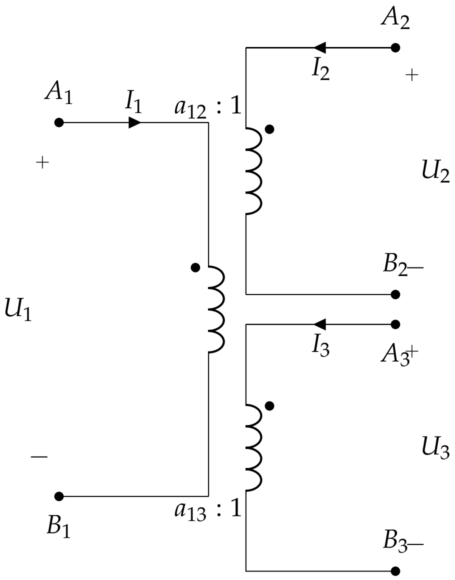
$$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 = a_{12}^2$$

$$\frac{L_1}{L_3} = \left(\frac{N_1}{N_3}\right)^2 = a_{13}^2$$

$$\frac{\overline{U}_1}{\overline{U}_2} = a_{12}$$

$$\frac{\overline{U}_1}{\overline{U}_3} = a_{13}$$

Transformador Ideal



Relación de Transformación del Transformador Ideal

Debido a las condiciones de idealidad:

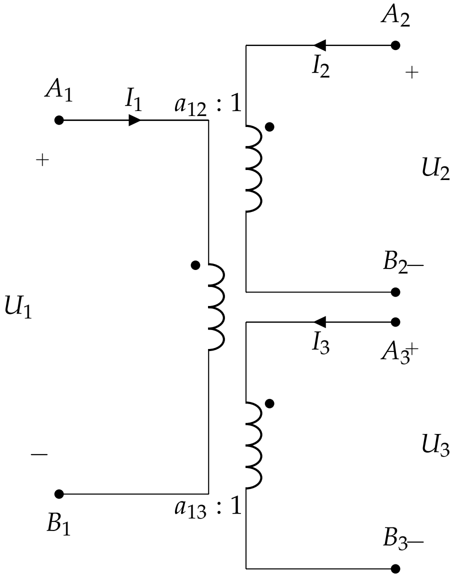
$$\overline{\phi}_{11} + \overline{\phi}_{22} + \overline{\phi}_{33} = 0$$

$$N_1\bar{I}_1 + N_2\bar{I}_2 + N_3\bar{I}_3 = 0$$

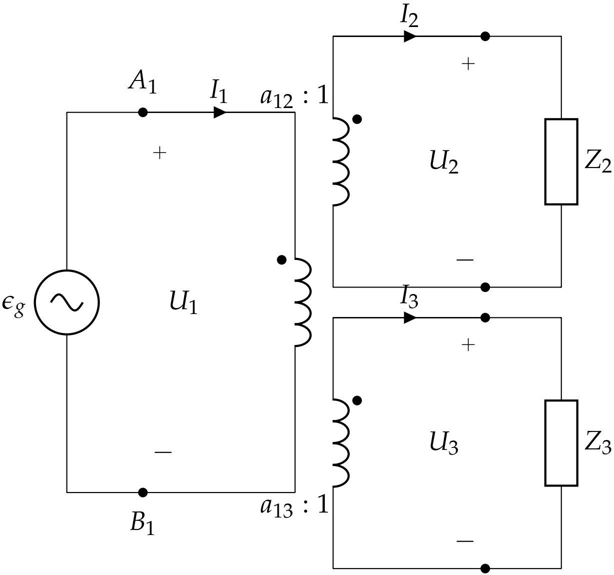
En términos de corriente:

$$\boxed{\bar{I}_1 = -1/a_{12} \cdot \bar{I}_2 - 1/a_{13} \cdot \bar{I}_3}$$

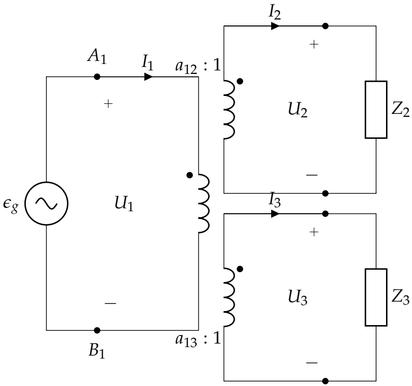
Si las corrientes I_2 e I_3 son salientes, en la ecuación anterior los signos serán positivos.



Impedancia de Entrada



Impedancia de Entrada*



Ecuaciones del transformador:

$$\overline{U}_1 = \overline{U}_2 \cdot a_{12}$$
$$\overline{U}_1 = \overline{U}_3 \cdot a_{13}$$
$$\overline{I}_1 = 1/a_{12} \cdot \overline{I}_2 + 1/a_{13} \cdot \overline{I}_3$$

Ecuaciones Terminales

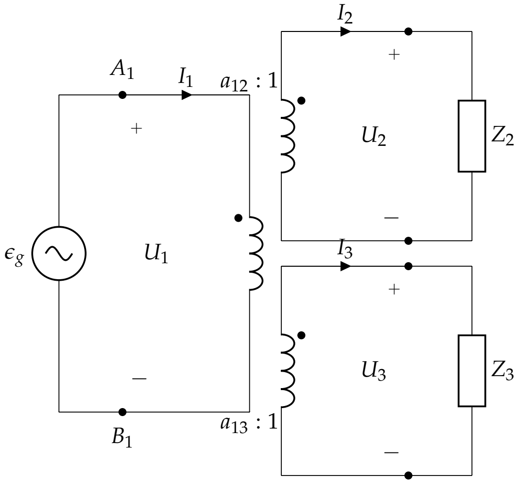
$$\overline{U}_2 = \overline{Z}_2 \cdot \overline{I}_2$$
$$\overline{U}_3 = \overline{Z}_3 \cdot \overline{I}_3$$

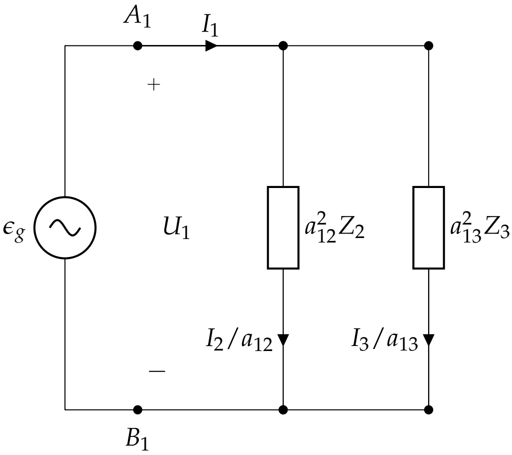
Resultado:

$$\frac{\overline{I}_1}{\overline{U}_1} = \boxed{\overline{Y}_{in} = \frac{1}{a_{12}^2 \overline{Z}_2} + \frac{1}{a_{13}^2 \overline{Z}_3}}$$

*Las corrientes en 2 y 3 son salientes, y por tanto, en la ecuación de corrientes usamos el signo positivo.

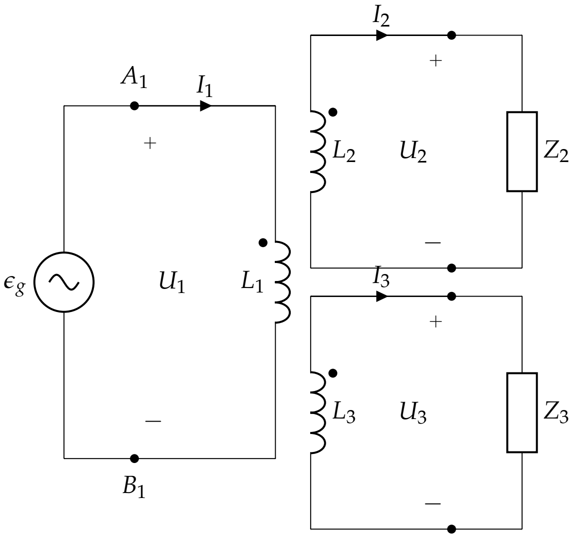
Circuito Equivalente



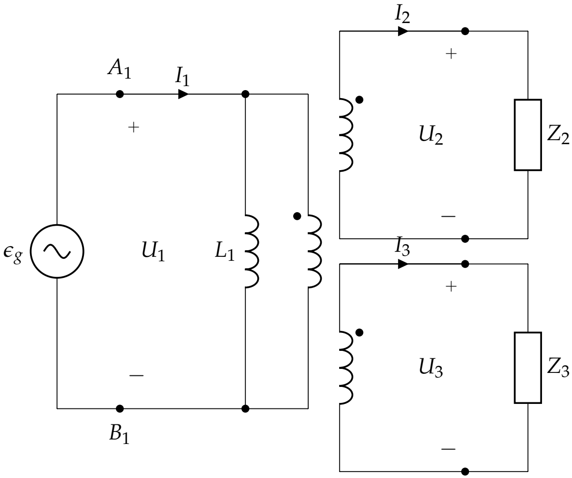


Circuito Equivalente de un Transformador Perfecto

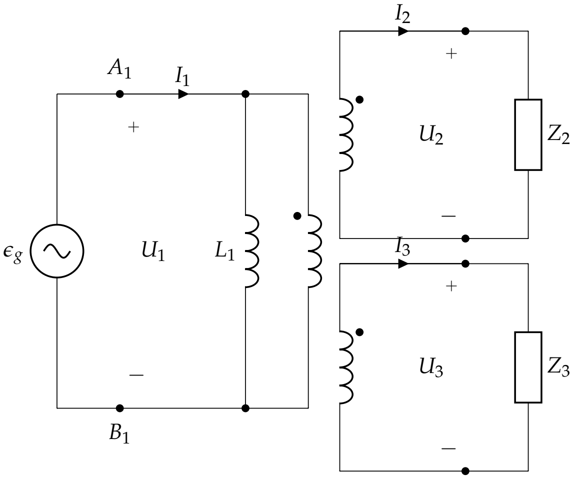
Transformador Perfecto

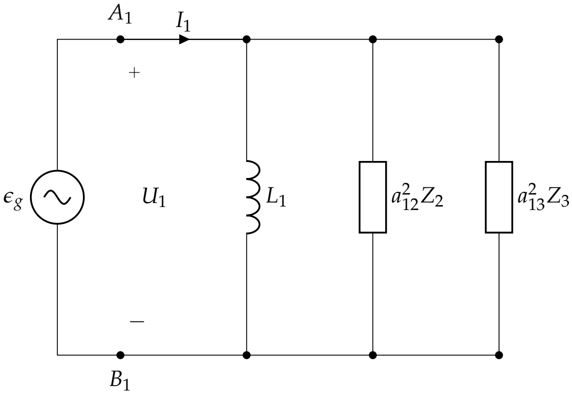


Equivalente Ideal

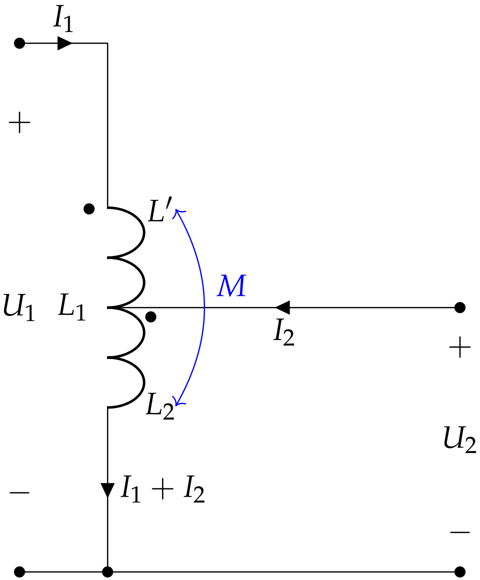


Circuito Equivalente de un Transformador Perfecto

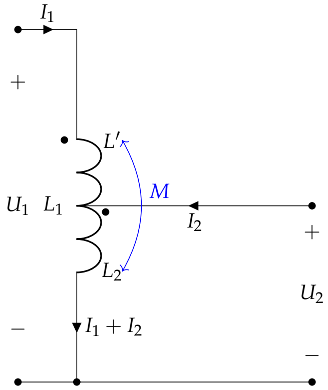




Autotransformador Perfecto



Ecuaciones del Autotransformador Perfecto

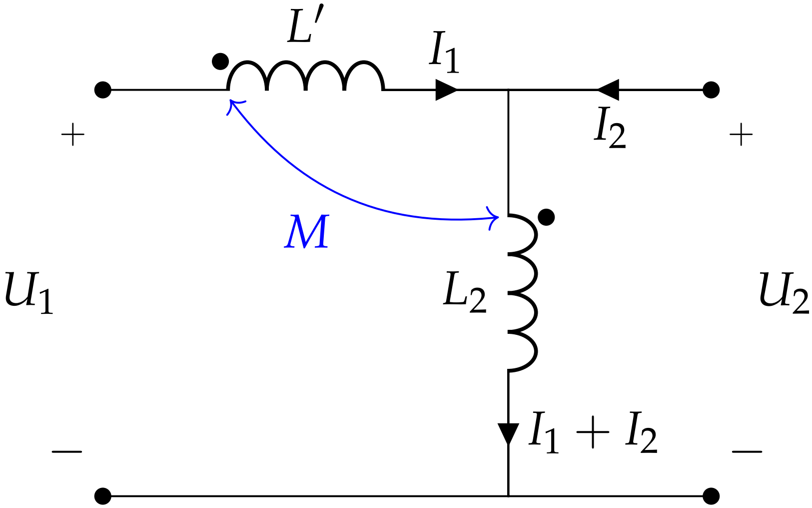


$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1 + j\omega(M + L_2) \cdot \bar{I}_2$$

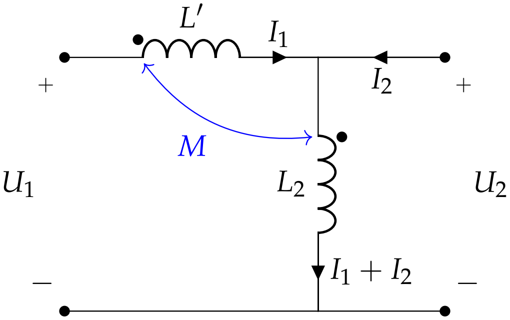
$$\overline{U}_2 = j\omega(M + L_2) \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

$$M = \sqrt{L' \cdot L_2}$$

Circuito Alternativo del Autotransformador Perfecto

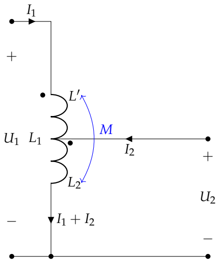


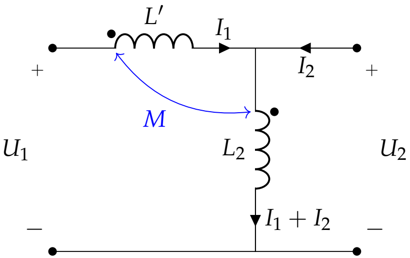
Ecuaciones del Circuito Alternativo



$$\overline{U}_1 = j\omega(L' + L_2 + 2M) \cdot \bar{I}_1 + j\omega(L_2 + M) \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega(L_2 + M) \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Ecuaciones Comparadas



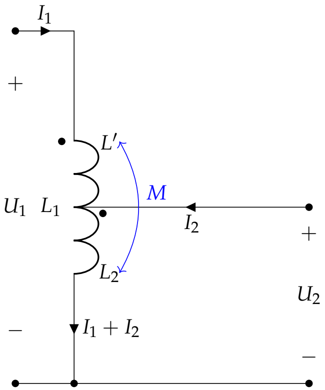


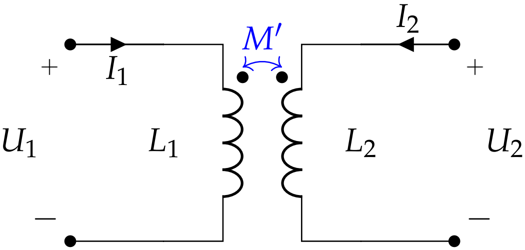
$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega (M + L_2) \cdot \overline{I}_2$$
$$\overline{U}_2 = j\omega (L_2 + M) \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2$$

$$\overline{U}_1 = j\omega (L' + L_2 + 2M) \cdot \overline{I}_1 + j\omega (M + L_2) \cdot \overline{I}_2$$
$$\overline{U}_2 = j\omega (L_2 + M) \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2$$

$$L_1 = L' + L_2 + 2M$$
$$M = \sqrt{L' \cdot L_2}$$

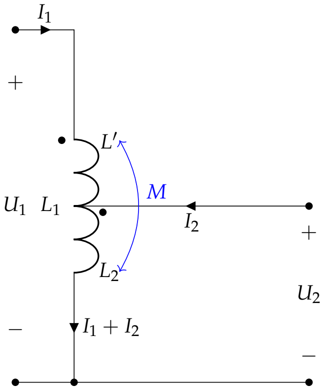
Transformador Perfecto Equivalente

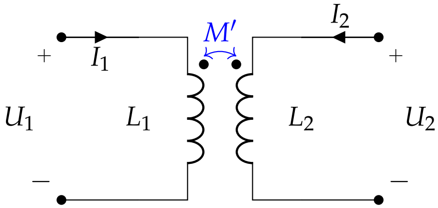




$$L_1 = L' + L_2 + 2M$$
$$M' = M + L_2$$

Transformador Perfecto Equivalente

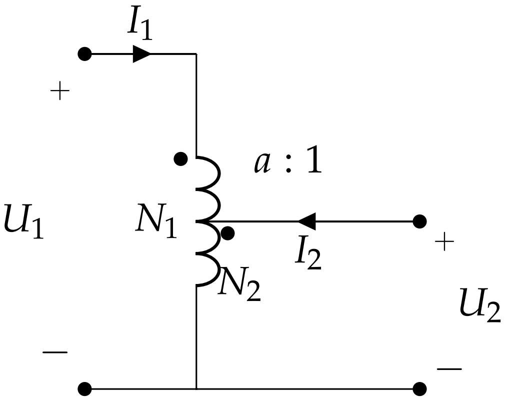


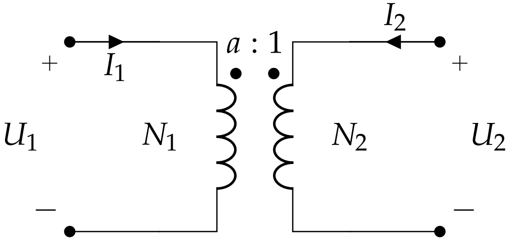


Comprobamos:

$$
\begin{aligned}
M' &= \sqrt{L_1 \cdot L_2} = \\
&= \sqrt{(L' + L_2 + 2M)L_2} = \\
&= \sqrt{L'L_2 + L_2^2 + 2ML_2} = \\
&= \sqrt{M^2 + L_2^2 + 2ML_2} = \\
&= M + L_2
\end{aligned}
$$

Equivalente del Autotransformador Ideal





$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = a$$

$$\frac{I_1}{I_2} = -\frac{N_2}{N_1} = -\frac{1}{a}$$